SOLUTION BY BENJAMIN F. YANNEY, College of Wooster, Wooster, Ohio.

Let the digits in the initial order be

$$a_n,\ a_{n-1},\ \cdots,\ a_1.$$

Then by hypothesis,

$$a_n 10^{n-1} + a_{n-1}10^{n-2} + \cdots + a_2 10 + a_1 \equiv 0 \pmod{p}.$$

Multiply each member of the congruence by 10, remembering that $10^n \equiv 1 \pmod{p}$, and place the digit $a_n$ in units place. We thus secure one cyclic permutation. Another cyclic permutation is secured by multiplying again by 10; and so on. This completes the proof.

It will be observed that the theorem may be generalized by multiplying all the terms to the left of any specified term by $10^{\kappa n}$, in the case of any cyclic permutation, where $\kappa$ is any positive integer. Thus, in the example given, 480001, 800000014, and 140000000008 are also each multiples of 37. By successive application of this method, we may obtain different types of cyclic permutations. Thus, 400000080001 is a multiple of 37. We may have other than cyclic permutations, with ciphers, by multiplying any one or more terms of the above congruence by $10^{\kappa n}$, where $\kappa$ can have a different value for each term multiplied. Thus, 80401 is also a multiple of 37. It is interesting to note in this more general application that no two integers of the original number can ever collide.

Also solved by L. C. MATHEWSON, PHILIP FRANKLIN, W. R. RANSOM, FRANK IRWIN, PAUL CAPRON, HORACE OLSON, and C. C. YEN.

**262 (Number Theory). Proposed by C. N. SCHMALL, New York City.**

If $x$, $y$, $z$, are 3 integers, consecutive among the integers prime to 3, show that

$$x(x - 2y) - z(z - 2y) = \pm 3.$$

SOLUTION BY EDWARD H. VANCE, Graduate Student, Ithaca, N. Y.

Let $v - 1$ be any number divisible by 3, then any set of three integers consecutive among the numbers prime to 3 may be represented by one of the following sets:

$$v - 2,\ v,\ v + 1;\quad v,\ v + 1,\ v + 3.$$

Substituting $v - 2$, $v$, $v + 1$ for $x$, $y$, $z$, respectively, in the lefthand side of the given equation we have

$$x(x - 2y) - z(z - 2y) = 3.$$

Substituting $v$, $v + 1$, $v + 3$ for $x$, $y$, $z$, respectively, we have

$$x(x - 2y) - z(z - 2y) = -3.$$

Also solved by PAUL CAPRON, N. P. PANDYA, LOUIS O'SHAUGHNESSY, LEWIS CLARK, E. F. CANADAY, GEORGE W. HARTWELL, J. L. RILEY, ALBERT G. RAU, HERBERT N. CARLETON, HORACE OLSON, and V. M. SPUNAR.

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# QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas.

## DISCUSSIONS.

### I. ON MAKING MATHEMATICAL RESULTS MORE AVAILABLE FOR ENGINEERS.

By WILLIS WHITED, Harrisburg, Pennsylvania.

Some time ago I received a circular from the Mathematical Association of America regarding the *Annals of Mathematics.* I like very much the idea of a

series of articles setting forth the "state of the art" of the different branches of mathematics in a form that would be intelligible to people who are not specialists in the respective branches.

I am an engineer and know that there are numerous unsolved problems in engineering science which are chiefly mathematical. The engineer studies mathematics primarily for its value as a tool in solving his problems, however fond he may be of the subject for its own sake. Very few engineers find time, in the course of an ordinary lifetime, to acquire a reasonably complete knowledge of all the pure mathematics that they can use to advantage in following up the latest advances in their respective specialties and in doing the research work that devolves upon them. It not infrequently happens that work which appears at the time to be little more than mathematical gymnastics is subsequently developed into something quite useful; but years elapse before the people who need the mathematics learn of its existence. The investigating engineer and the mathematician must keep in closer touch with each other in the future than they have in the past. America must take a larger place in the advancement of science.

The engineering investigator who encounters difficult mathematical problems must have better facilities for acquiring the knowledge he needs of the many powerful methods of mathematical analysis which have been developed within the memory of men now living. Works on advanced mathematics are practically all intended for professional mathematicians. Their contents are almost wholly academic in character and they are beyond the reach of the engineer. Articles in mathematical periodicals are seldom intelligible to any but a very few specialists. This is doubtless unavoidable and perfectly proper, but I would urge that occasional articles be written bringing various branches of the subject down to date, omitting, perhaps, much of the purely academic work and expressing the whole, if possible, in terms that can be understood by the engineer who has kept up his collegiate mathematics.

From what little I know of modern mathematics, I would imagine that progress useful to the engineer has been or soon may be attained in the following branches (among others): differential equations, calculus of finite differences, vector analysis, successive integration, elliptic and hyperelliptic functions, transcendental equations and analytical geometry.

Most of the modern writers on advanced analytical geometry use homogeneous coördinates. This method has some advantages in certain kinds of work, but it is rarely taught to undergraduates in engineering and, moreover, most of the engineer's problems are metrical, so that Cartesian coördinates are better adapted to their solution. Many theorems in projective geometry could be used by the engineer who employs graphical solutions if the theorems were put in such form that he could acquire a knowledge of them in a reasonable time.

Most of the fundamental principles of those branches of science which aspire to become exact can best be expressed in the form of differential equations. Many of these equations have not, thus far, been solved. Approximate solutions are better than none. Hence, I would urge that methods of approximate solu-

tions be so developed as to make them, so far as practicable, accessible to the engineer. In the practical applications of mathematics to engineering and, probably, to other sciences, the solutions of problems are often not exact. Graphical solutions are subject to a very considerable margin of error and arithmetical solutions almost always involve the multiplication or division of decimals in which only a certain number of decimal places are retained. Transcendental functions and radicals are only given approximately in the tables and it may well happen that a solution in a rapidly converging series is just as convenient as an exact solution. If a solution is in the form of a series with general expressions for coefficients, it may be almost as satisfactory as any other kind of a formula. In that case, if a similar problem occurs again, it will only be necessary to substitute the proper values for the constant terms in the coefficients, which can be done by an assistant who is not familiar with differential equations. I therefore hope that mathematicians will publish freely their methods for approximate solutions of differential equations and other problems, preferably in a form that will not compel the busy engineer to search through a multitude of monographs, many of which are in foreign languages and some of which can not be readily obtained, before he can get an adequate idea of the nature of the solution.

Elliptic integrals are met with occasionally and if they merely have to be integrated once approximate methods are available. If successive integration is required, it is apt to be "another story."

It may be that all problems that can be solved by vector analysis can also be solved by the older methods, but this method is often so much simpler that the subject is worthy thorough study.

The engineer often meets with transcendental equations and they usually have to be solved as individual problems. If more general methods, even if only approximate, have been developed, they should be more generally known.

Complex variables are occasionally encountered, chiefly in connection with differential equations. If a practical knowledge of the subject could be imparted without requiring the reader to toil through ponderous tomes in an effort to find an explanation, it would be helpful.

The modern theory of functions is a subject which is very interesting to one who is fond of mathematics for its own sake; but can not some way be found by which the student can get at the pith of the matter in a reasonable time? The subject is chiefly academic, but is very attractive.

## II. Relating to New Remainder Terms for Certain Integration Formulæ.

By S. A. Corey, Albia, Iowa.

In the June, 1917, number of the Monthly Professor Daniell notes the fact that at least one of the remainder terms of the integration formulæ which I gave in the June–July, 1912, number of the Monthly is needlessly large. I also observe that the remainder term to my formula 25*s* which he gives is too small, as he has tacitly made the unwarranted assumption that the signs of his $S_1$ and $S_2$